Truncation of a 2-dimensional Fermi surface
due to quasiparticle gap formation at the saddle points

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We study a two-dimensional Fermi liquid with a Fermi surface containing the saddle points $(\pi, 0)$ and $(0, \pi)$. Including Cooper and Peierls channel contributions leads to a one-loop renormalization group flow to strong coupling for short range repulsive interactions. In a certain parameter range the characteristics of the fixed point, opening of a spin and charge gap and dominant pairing correlations are similar to those of a 2-leg ladder at half-filling. An increase of the electron density we argue leads to a truncation of the Fermi surface with only 4 disconnected arcs remaining.

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The origin of the instability of the Landau-Fermi liquid state as the electron density is increased in overdoped cuprates is one of the most interesting open questions in the field. Recently we proposed that the origin lies in a flow of Umklapp scattering to strong coupling [1]. The simpler case with the Fermi surface (FS) extrema at $(\pm \pi/2, \pm \pi/2)$ was considered and not the realistic case for hole-doped cuprates where the leading contribution from Umklapp processes comes from scattering at the saddle points $(\pi, 0)$ and $(0, \pi)$. In this letter we report a one-loop renormalization group (RG) calculation for the realistic case including contributions from both Cooper and Peierls channels. Reasonable conditions can lead to a strong coupling fixed point whose characteristics are similar to those of half-filled 2-leg ladders. There strong coupling Umklapp processes lead to spin and charge gaps but only short range spin correlations. A particularly interesting and novel feature is that although the strongest divergence is in the d-wave pairing channel, the charge gap causes insulating not superconducting behavior.

There have been a number of previous RG investigations for a FS with saddle points. Schulz [2] and Dzyaloshinskii [3] considered the special case with only nearest neighbor (n.n.) hopping so that the saddle points coincides with a square FS and perfect nesting exactly at half-filling, leading to a fixed point with long range antiferromagnetic (AF) order. Lederer *et al.* [4] and Dzyaloshinskii [5] also considered the same model as we do. There are two fixed points, one at a strong coupling fixed point with d-wave pairing found by Lederer *et al.* [4], and a weak coupling examined by Dzyaloshinskii

[5]. A Hubbard parametrization of the repulsive interactions (U) and moderate interaction strength suffices to stabilize the strong coupling fixed point. The new feature we wish to stress is that there can be both spin and charge gaps. The FS is then truncated through the formation of an insulating spin liquid (ISL) with resonance valence bond (RVB) character. We propose that as the hole doping decreases these gaps spread out from the saddle points so the FS consists of a set of arcs, which progressively shrink as the hole doping decreases.

We start with a 2-dimensional FS touching the saddle points $(\pi, 0)$ and $(0, \pi)$. Such a FS is realized in the case of the dispersion relation $\varepsilon(\mathbf{k}) = -2t(\cos k_x + \cos k_y) - 4t' \cos k_x \cos k_y$ with $t > 0$ ($t' < 0$) as n.n. (n.n.n.) hoppings. Throughout this letter, we assume t'/t small but nonzero so that we are close to half-filling. Due to the van Hove singularity, the leading singularity arises from electron states in the vicinity of the saddle points. We consider two FS patches at the saddle points and examine the coupling between them using one-loop RG equations, as illustrated in Fig. 1a. k_c is the radius of the patches.

The susceptibility for the Cooper channel at $q = 0$ has a log-square behavior of the form

$$\chi_0^{\mathrm{pp}}(\omega) = -h \ln(\omega/E_0) \ln(\omega/2tk_c{}^2). \qquad (1)$$

Here, the sum over k is restricted to the patches. E_0 is the cutoff energy and $h = (8\pi^2 t)^{-1}$ for $|t'/t| \ll 1$. The Peierls channel at $Q = (\pi, \pi)$ diverges as

$$\chi_Q^{\mathrm{ph}}(\omega) = \begin{cases} h \ln(\omega/E_0) \ln(\omega/2tk_c{}^2) & \omega \gg |t'| \\ 2h \ln |t'/t| \ln(\omega/E_0) & \omega \ll |t'| \end{cases} . \quad (2)$$

The susceptibilities for the Peierls channel at $q = 0$ (χ_0^{ph}) and the Cooper channel at $q = Q$ (χ_Q^{pp}) also diverges

$$\chi_0^{\mathrm{ph}} \sim -\chi_Q^{\mathrm{pp}} \sim 2h \ln(E_0/\omega), \qquad (3)$$

but the coefficients of $\ln(\omega)$ are smaller than that of χ_Q^{ph}.

In Fig. 2 we define the interaction vertices g_i ($i = 1 \sim 4$). Normal and Umklapp processes are indistinguishable since the patches are at the zone edge. We use a Wilson RG flow, parametrized by a decreasing energy scale, in which all degrees of freedom above that energy scale are integrated out. Wilson's effective action at scale E_0 has

the dual interpretation that (a) it generates the interaction vertices, and thus an effective Hamiltonian, for the particles with energy below E_0 and (b) these vertices are also the connected correlation functions with the infrared cutoff E_0. We consider only the four-point function and include only the one-loop terms. The one-loop RG was justified as the leading behavior at low energies and weak coupling for a class of FS in ref. [6], which includes those with nonzero curvature ($t' \neq 0$ in our case). It leads to the flow equations (see also Lederer *et al.* [4])

$$\dot{g}_1 = 2d_1 g_1(g_2 - g_1) + 2d_2 g_1 g_4 - 2d_3 g_1 g_2, \quad (4)$$

$$\dot{g}_2 = d_1(g_2{}^2 + g_3{}^2) + 2d_2(g_1 - g_2)g_4 - d_3(g_1{}^2 + g_2{}^2), \quad (5)$$

$$\dot{g}_3 = -2g_3 g_4 + 2d_1 g_3(2g_2 - g_1), \quad (6)$$

$$\dot{g}_4 = -(g_3{}^2 + g_4{}^2) + d_2(g_1{}^2 + 2g_1 g_2 - 2g_2{}^2 + g_4{}^2). \quad (7)$$

Here we introduced the normalization $g_i \rightarrow h g_i$ to give dimensionless couplings, and $\dot{g}_i \equiv (\mathrm{d}g_i)/(\mathrm{d}y)$ where $y \equiv \ln^2(\omega/E_0) \propto \chi_0^{\mathrm{pp}}(\omega)$. We define functions which describe the relative weight of $q = 0$ Cooper channel contribution and those of other channels

$$d_1(y) = \mathrm{d}\chi_Q^{\mathrm{ph}}/\mathrm{d}y, \quad (8)$$

$$d_2(y) = \mathrm{d}\chi_0^{\mathrm{ph}}/\mathrm{d}y, \quad (9)$$

$$d_3(y) = -\mathrm{d}\chi_Q^{\mathrm{pp}}/\mathrm{d}y. \quad (10)$$

Their asymptotic forms are $d_1(y) \rightarrow 1$ at $y \approx 1$ and $d_1(y) \sim \ln|t/t'|/\sqrt{y}$ as $y \rightarrow \infty$, while $d_2(y) \sim d_3(y) \sim 1/\sqrt{y}$ throughout the region of interest.

The case $d_1 = 1$ and $d_2, d_3 \ll d_1$ was studied by Schulz [2], Dzyaloshinskii [3] and Lederer *et al.* [4] which arises at $t' = 0$ as well as in a sufficiently large U region where t' is irrelevant. SDW susceptibility has the same exponent as d-wave pairing but is dominant due to the next leading divergent terms. The fixed point is understood as a Mott insulator with long range AF order. The limit $d_1 = d_2 = d_3 = 0$ was treated by Dzyaloshinskii [5]. In this case (6) and (7) combine to give $\dot{g}_- = -g_-{}^2$ with $g_- = g_4 - g_3$. Dzyaloshinskii considered the weak-coupling fixed point $g_- \rightarrow 0$ which arises when $g_- \geq 0$, and discussed the resulting Tomonaga-Luttinger liquid behavior.

In this letter we examine the RG equations with $0 < d_1(y) < 1$ which enables us to consider nonzero values of the ratios t'/t and U/t. Since $d_2, d_3 \ll d_1$, we neglect d_2 and d_3 in RG equations for simplicity. Note the terms involving d_1 act to enhance the basin of attraction for the strong coupling fixed point, $g_- \rightarrow -\infty$ [4]. The one-loop RG equations are solved numerically. Starting from a Hubbard-model initial value $g_i = U$ ($i = 1 \sim 4$), the vertices flow to strong coupling fixed points with $g_2 \rightarrow +\infty$, $g_3 \rightarrow +\infty$ and $g_4 \rightarrow -\infty$, with the asymptotic form

$$g_i(y) = g_i^0/(y_c - y). \quad (11)$$

Here $y_c \sim t/U$ is the critical point of one-loop RG equations. The divergence of $g_1(y)$ with respect to $y_c - y$ is only logarithmic. To analyze this fixed point more precisely, we substitute the asymptotic form (11) into eqs. (4)-(7) and obtain polynomial equations

$$g_1^0 = 2d_1(y_c) \cdot g_1^0(g_2^0 - g_1^0), \quad (12)$$

$$g_2^0 = d_1(y_c) \cdot ((g_2^0)^2 + (g_3^0)^2), \quad (13)$$

$$g_3^0 = -2g_3^0 g_4^0 + 2d_1(y_c) \cdot g_3^0(2g_2^0 - g_1^0), \quad (14)$$

$$g_4^0 = -((g_3^0)^2 + (g_4^0)^2) \quad (15)$$

Fig. 3 shows the solution of these equations g_i^0 for the initial values $g_i = U$. The coefficients g_i^0 are determined as a function of $d_1(y_c) \sim \sqrt{U/t} \ln|t/t'|$, i.e. the critical behavior of the fixed point is a function of U.

Although one cannot solve for the strong coupling fixed point using only one-loop RG equations, a qualitative description comes from the susceptibilities. Using these coefficients g_i^0, exponents for various susceptibilities are calculated as follows. The one-loop RG eqn. for the d-wave pairing is

$$\dot{\bar{\chi}}_{\mathrm{dP}} = 2(g_4 - g_3), \quad (16)$$

where $\bar{\chi}_{\mathrm{dP}} = (\partial \chi_{\mathrm{dP}}/\partial \omega)/(\partial \chi_0^{\mathrm{pp}}/\partial \omega)$. From eq. (11), we obtain a divergence $\chi_{\mathrm{dP}} \propto (y_c - y)^\alpha$ with exponent $\alpha = \alpha_{\mathrm{dP}} = 2(g_4^0 - g_3^0)$. Similarly, exponents for s-wave pairing, charge (CDW), spin (SDW) density waves as well as uniform spin, charge compressibilities and finite momentum π-pairing are given by

$$\alpha_{\mathrm{sP}} = 2(g_3^0 + g_4^0), \quad (17)$$

$$\alpha_{\mathrm{CDW}} = (2g_1^0 - g_2^0 + g_3^0) \, d_1(y_c), \quad (18)$$

$$\alpha_{\mathrm{SDW}} = -2(g_2^0 + g_3^0) \, d_1(y_c), \quad (19)$$

$$\alpha_{\mathrm{s}} = -2(g_1^0 + g_4^0) \, d_2(y_c), \quad (20)$$

$$\alpha_\kappa = (-g_1^0 + 2g_2^0 + g_4^0) \, d_2(y_c), \quad (21)$$

$$\alpha_\pi = 2(-g_1^0 + g_2^0) \, d_3(y_c), \quad (22)$$

respectively. For weak coupling, we have $d_2(y_c) \sim d_3(y_c) \sim \sqrt{U/t}$. Uniform susceptibilities are calculated in the limit $\omega, q \rightarrow 0$ with q/ω held fixed.

In Fig. 4 we show the exponents for d-wave pairing, SDW, uniform spin and charge compressibility. Comparison of the values of the exponents shows us that the most divergent susceptibility is d-wave pairing throughout the parameter region of $0 < d_1(y_c) < 1$ [4]. The SDW susceptibility shows a weaker divergence and the exponent vanishes in the limit $d_1(y_c) \rightarrow 0$ or $U/t \rightarrow 0$. The exponents for uniform spin as well as s-wave pairing, CDW and π-pairing are always positive, i.e. these susceptibilities are suppressed at low frequency.

The exponent for the charge compressibility changes sign at $d_1(y_c) \sim 0.6$. Namely there exists a critical interaction strength U_c such that for $U > U_c$ the charge compressibility is suppressed to zero. The critical value U_c is determined by t' in the form $U_c/t \propto \ln^{-2}|t/t'|$. This implies a transition from a superconducting phase

at $U < U_c$ with its origin in enhanced Cooper pairing due to the van Hove singularity, to a charge-gapped phase at $U > U_c$ which can be regarded as a precursor of the Mott transition. The fixed point at $U > U_c$ resembles that of the half-filled 2-leg ladder which has spin and charge gaps but the most divergent susceptibility is d-wave pairing. This fixed point (C0S0 in the Balents-Fisher [7] notation) is well understood as an ISL of short range RVB form. The close similarity between the fixed points leads us to assign them to the same universality class.

The fixed point for $0 < d_1 < 1$ with d-wave pairing as the leading divergence differs from the case $d_1 \equiv 1$ where SDW correlation is dominant when the next leading term is included [2], and from the weak-coupling fixed point for $d_1 \equiv 0$ [5]. The contribution of the particle-particle channel of the part of the Fermi surface away from the saddle points was not discussed here for brevity; one can show that it gives a nonnegative contribution to $\dot{g}_-$ and thus even enhances the asymmetry that drives the flow to strong coupling $g_- \rightarrow -\infty$. The present result for the saddle point model with $t' < 0$ is in good accordance with the case $t' > 0$ (or, $t' < 0$ with electron doping) previously studied by the authors [1]. There, 4 patches at the zone diagonals were considered. For interaction U larger than a critical value determined by the infrared cutoff due to finite curvature, the charge compressibility renormalizes to zero due to the Umklapp scattering.

Next we consider increasing the electron density. One possibility is to follow the non-interacting FS which expands beyond the saddle points. But the flow to strong coupling and the opening of a charge and spin gap leads us to consider a second possibility, namely that the FS is pinned by Umklapp processes and does not expand beyond the saddle points. This proposal was put forward in Ref. [1] after an examination of 8 FS-patches located on the Umklapp surface (US) which is defined by lines joining saddle points. The leading contribution from Umklapp processes comes from scattering between points on this US. Support for this proposal comes from the lightly doped 3-leg ladder [8,9] where in strong coupling a C1S1 phase occurs with an ISL with exactly half-filling in the even parity channels and an open FS only in the odd parity channel. This contrasts with the one-loop RG results which gives a C2S1 phase [7] with holes immediately entering both odd and even parity channels. Our proposal, sketched in Fig. 1b, is based on a lateral spread of the spin and charge gaps along the US leading to 4 open FS segments consisting of arcs centered at the points $(\pm\pi/2, \pm\pi/2)$. Such behavior can be viewed as a sort of phase separation in $\vec{k}$-space in that in some directions an ISL forms but others remain metallic. Note the area enclosed by the surface defined by the US and these 4 arcs contains the full electron density, consistent with a generalized form of Luttinger's Theorem.

Note since the ISL is not characterized by any simple broken symmetry or order parameter, the resulting state cannot be described by a simple mean field or Hartree-Fock factorization. Our proposal of a FS consisting of 4 disconnected arcs has strong parallels to recent gauge theory calculations for the lightly doped strong coupling t-J model by Lee and Wen [10]. Signs of such behavior are also evident in a recent analysis of the momentum distribution using a high temperature series by Putikka *et al.* [11]. Note models which include only n.n. hoppings ($t' = 0$) are a special limit from the present point of view.

The development of ISL near the saddle points is related to the gap formation of the high-T_c cuprates. In the normal state of the underdoped cuprates, the ARPES experiments [12,13] show a single particle gap opening and a loss of quasiparticle weight in the vicinity of the saddle points below the pseudo-gap temperature. Tunneling experiment [14] also shows a quasi-particle gap formation above T_c. These results are quite similar to those we propose in Fig. 1b. Systematics of the loss of quasiparticle weights in electron- and hole-doped cuprates [15] are also consistent with our results for 4-patch and 2-patch models.

The proposal that an ISL truncates the FS along the US in the vicinity of the saddle points has very interesting consequences. There will be a coupling to the open segments in the Cooper channel through the scattering of electron pairs out of the ISL to the open FS segments. This process is reminiscent of the coupling of fermions to bosonic preformed pairs in the Geshkenbein-Ioffe-Larkin model [16]. They argued for an infinite mass for such pairs to suppress their contribution to transport properties. Such scattering processes will be an efficient mechanism for d-wave pairing on the open FS segments.

We also see a close similarity to a phenomenological model by Ioffe and Millis [17], to explain the anomalous transport properties in the normal state. They assumed the FS segments to have usual quasi-particle properties without spin-charge separation but the scattering rate to vary strongly along the FS arcs. They justified the model by comparison to ARPES and tunneling experiments [12–14]. In our case, the scattering rate will vary strongly due to the strong Umklapp scattering at the end of the FS arcs where they meet the ISL region. Lastly we refer the reader to the recent paper by Balents, Fisher and Nayak [18] which introduces the concept of a Nodal liquid with properties similar to the ISL discussed above.

In conclusion we have shown that when the Fermi surface approaches the saddle points, Umklapp scattering drives the system into a strong coupling fixed point which can cause a breakdown of Landau Fermi liquid state. The Fermi surfaces near the saddle points $(\pi, 0)$ and $(0, \pi)$ are truncated by the formation of a pinned and insulating condensate, while the zone diagonal regions around $(\pm\pi/2, \pm\pi/2)$ remain metallic. We have given arguments that the spin properties are those of an insulating spin liquid. This microscopic model has a lot in common with

the results of ARPES experiments and some recent phenomenological models so that we believe it can form the basis for a theory of the cuprates.

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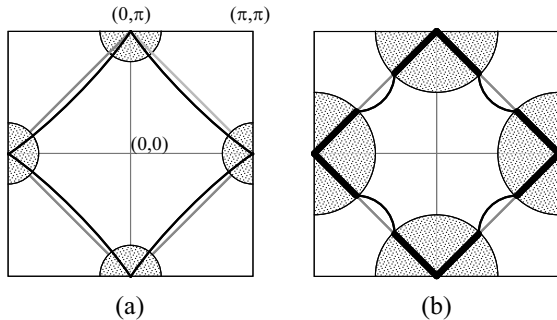
FIG. 1. Fermi surface (FS). (a) Two patches of the FS at the saddle points. (b) Truncated FS as electron density is increased.
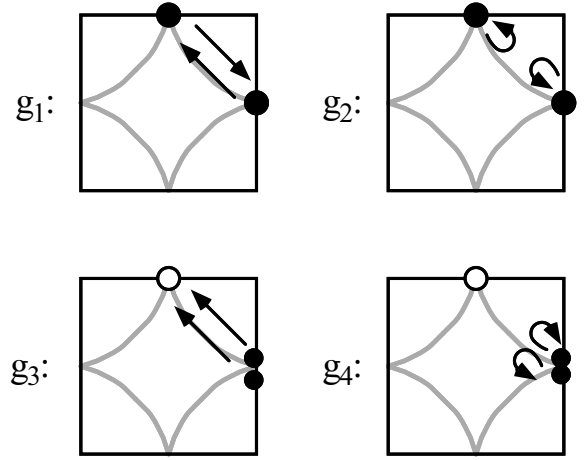

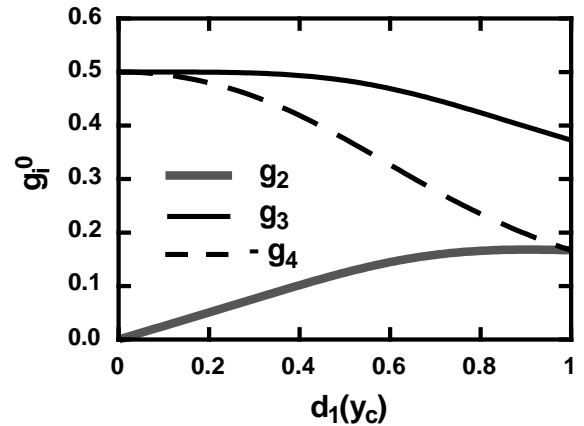FIG. 2. The definitions of vertices for the 2-patch model.

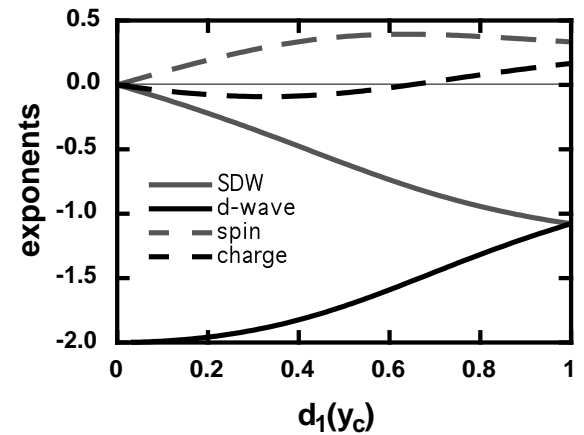FIG. 3. The fixed point values for g_i^0.

FIG. 4. Exponents for various susceptibilities. For uniform spin and charge susceptibilities, exponents are scaled by d_2/d_1